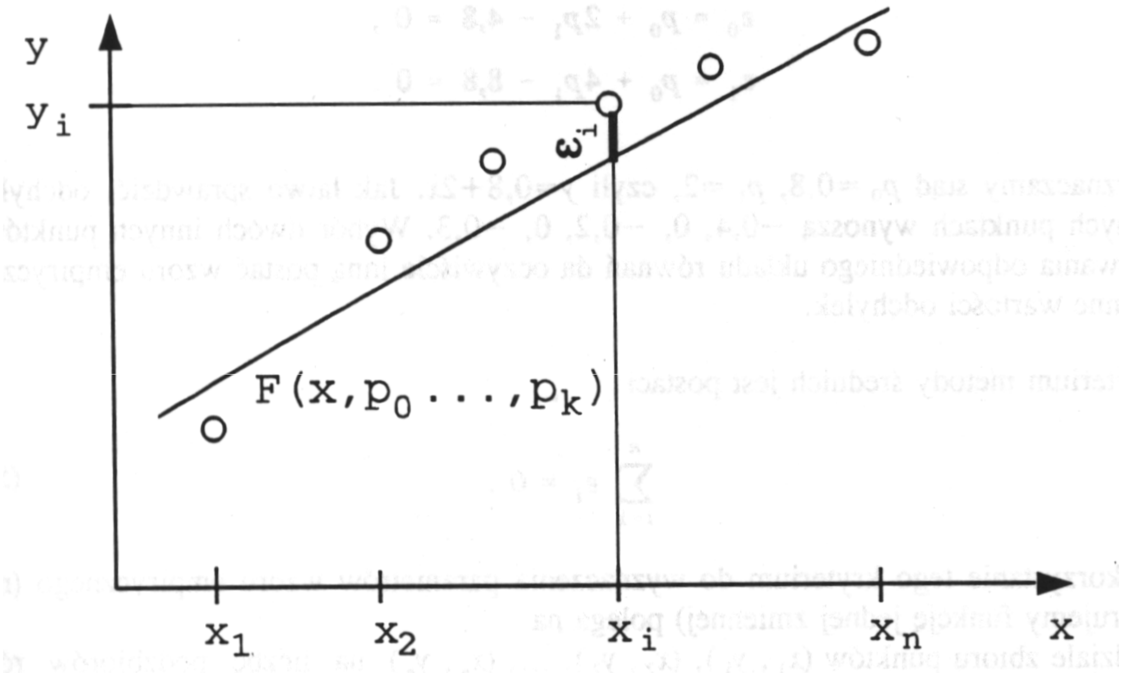
# Aproksymacja




$$(x_1, y_1),\quad (x_2, y_2) \quad \rightarrow \quad y = p_o + p_1 x$$

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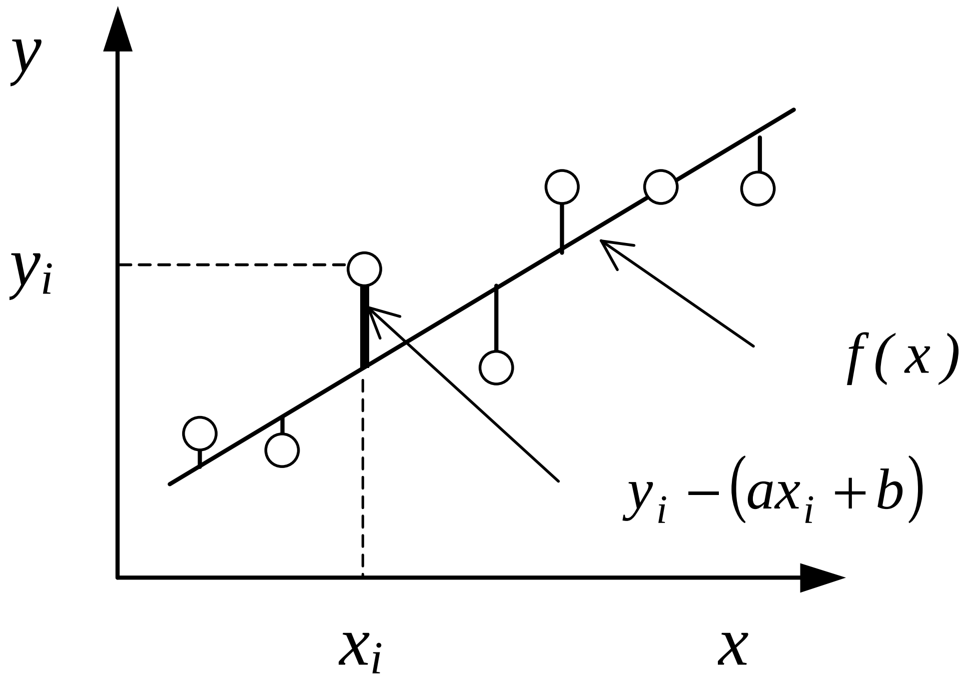



$$\rho = \sum_{i=1}^{m} r_i = \sum_{i=1}^{m} \left[ y_i - (ax_i + b) \right]^2$$

$$\sum_{i=1}^{m} 2(-x_i)\left[ y_i - (ax_i + b) \right] = 0$$

$$\sum_{i=1}^{m} 2(-1)\left[ y_i - (ax_i + b) \right] = 0$$

$$\left( \sum_{i=1}^{m} x_i^{\,2} \right) a + \left( \sum_{i=1}^{m} x_i \right) b = \sum_{i=1}^{m} x_i y_i$$

$$\left( \sum_{i=1}^{m} x_i \right) a + mb = \sum_{i=1}^{m} y_i$$

$$\begin{bmatrix} \sum_{i=1}^{m} x_i^{\,2} & \sum_{i=1}^{m} x_i \\ \sum_{i=1}^{m} x_i & m \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{m} x_i y_i \\ \sum_{i=1}^{m} y_i \end{bmatrix}$$

# Aproksymacja

$$\begin{bmatrix} \sum_{i=1}^{m} x_i^{\,2} & \sum_{i=1}^{m} x_i \\ \sum_{i=1}^{m} x_i & m \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{m} x_i y_i \\ \sum_{i=1}^{m} y_i \end{bmatrix}$$

$$\boldsymbol{A} = \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \vdots \\ x_m & 1 \end{bmatrix}$$

$$\begin{bmatrix} \sum_{i=1}^{m} x_i^{\,2} & \sum_{i=1}^{m} x_i \\ \sum_{i=1}^{m} x_i & m \end{bmatrix} = \begin{bmatrix} x_1 & x_2 & \cdots & x_m \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \vdots \\ x_m & 1 \end{bmatrix} = \boldsymbol{A}^T \boldsymbol{A}$$

$$\begin{bmatrix} \sum_{i=1}^{m} x_i y_i \\ \sum_{i=1}^{m} y_i \end{bmatrix} = \begin{bmatrix} x_1 & x_2 & \cdots & x_m \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix} = \boldsymbol{A}^T \boldsymbol{y}$$

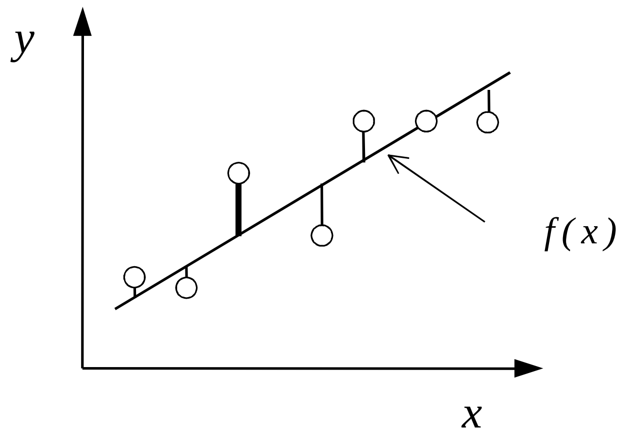
# Aproksymacja

$$\boldsymbol{A} = \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \vdots \\ x_m & 1 \end{bmatrix} \qquad \boldsymbol{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix} \qquad \boldsymbol{c} = \begin{bmatrix} a \\ b \end{bmatrix}$$

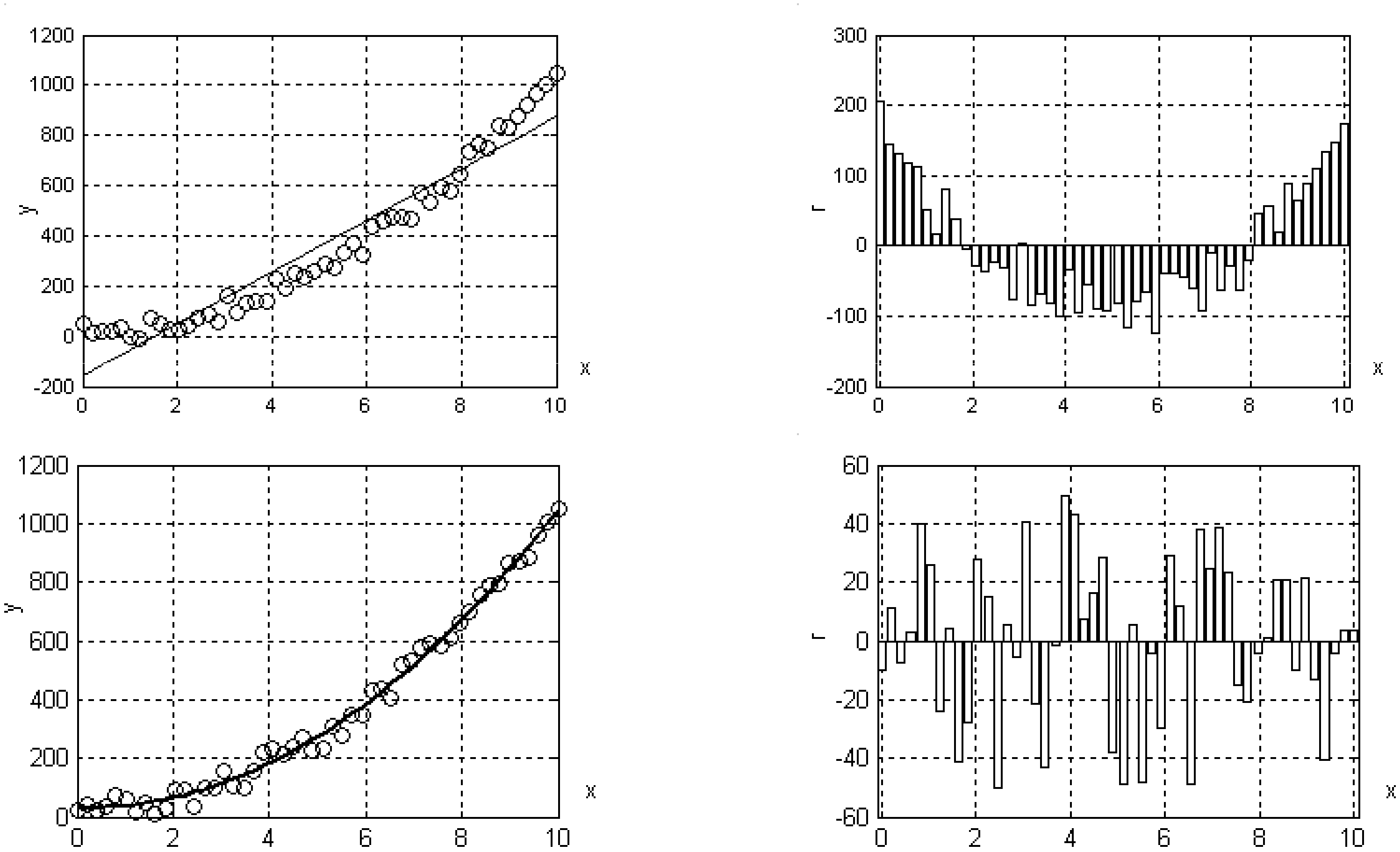
$$\left(\boldsymbol{A}^T \boldsymbol{A}\right)\boldsymbol{c} = \boldsymbol{A}^T \boldsymbol{y}$$

$$\boldsymbol{c} = \left(\boldsymbol{A}^T \boldsymbol{A}\right)^{-1} \boldsymbol{A}^T \boldsymbol{y}$$

$$f(x) = ax + b$$

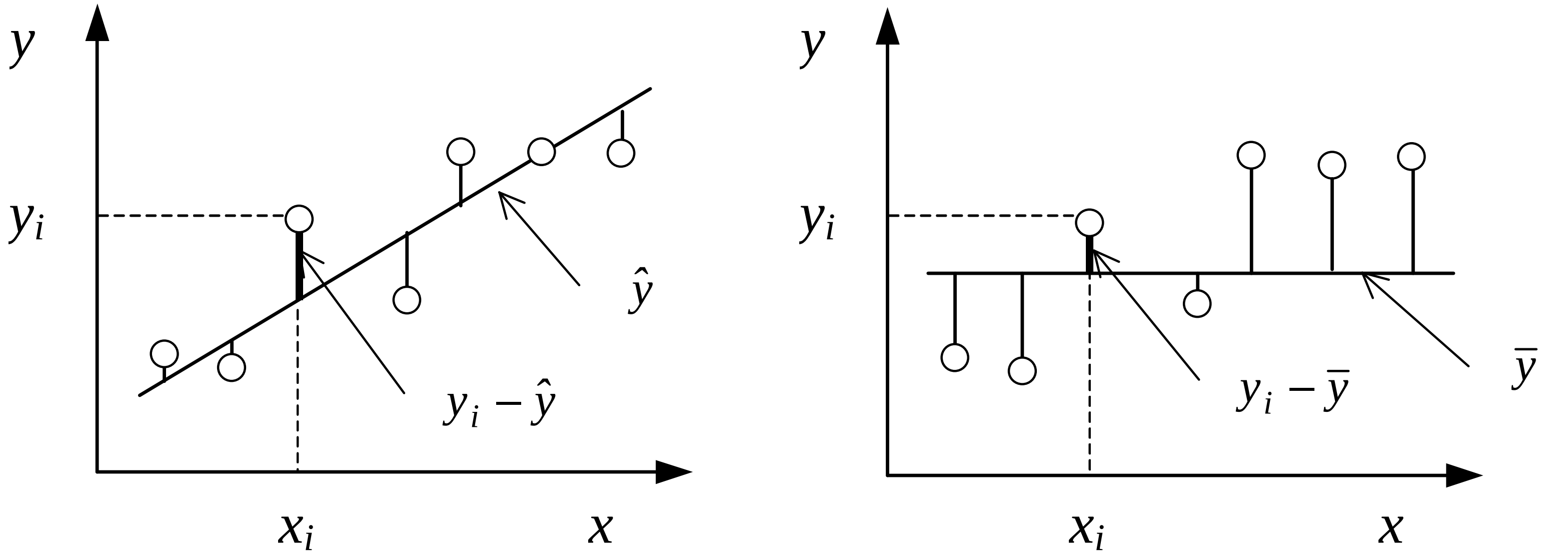


# Aproksymacja



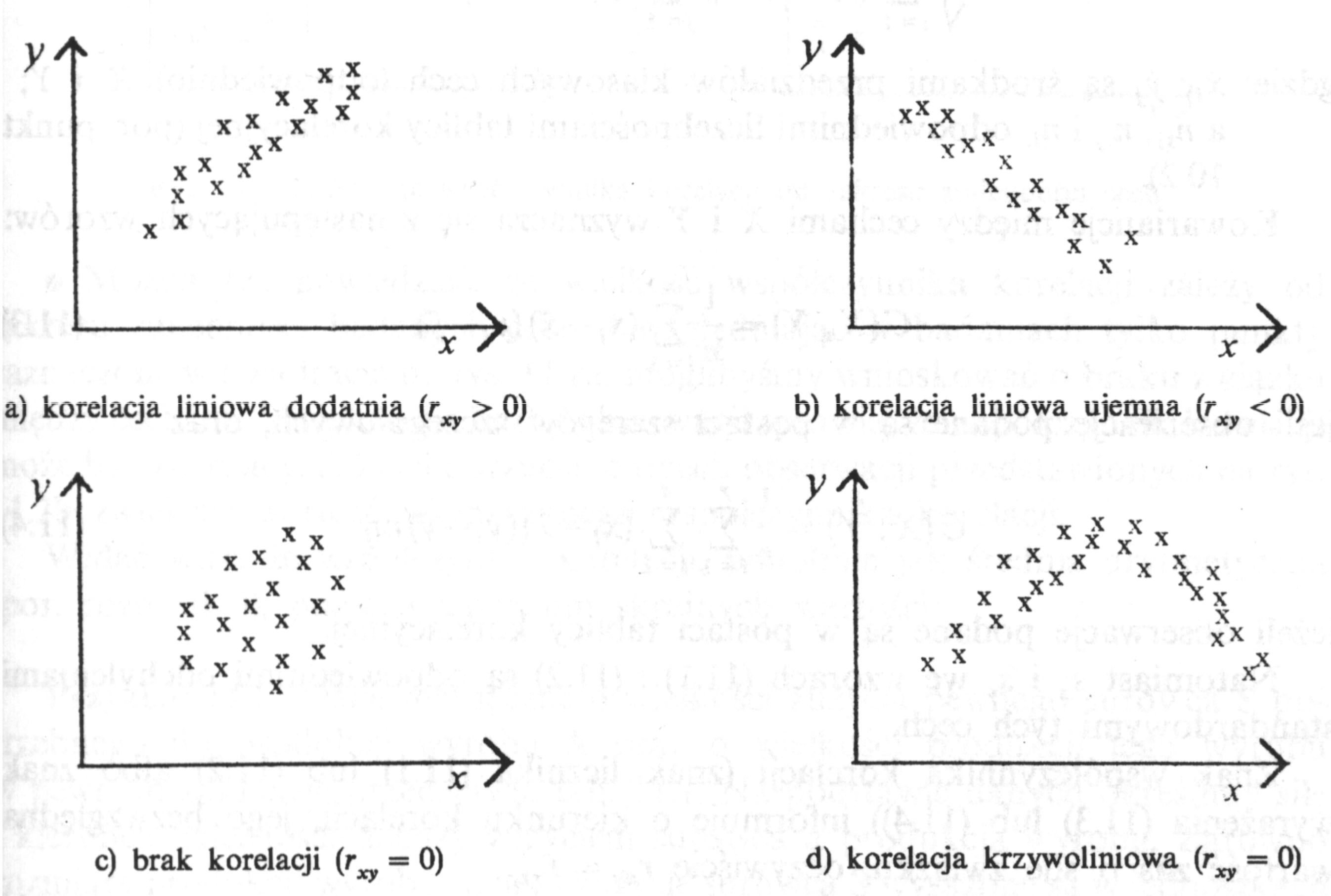

Ocena jakości aproksymacji – wektor reszt

# Aproksymacja




$$R^2 = 1 - \frac{\sum (y_i - \hat{y})^2}{\sum (y_i - \bar{y})^2}$$

# Aproksymacja



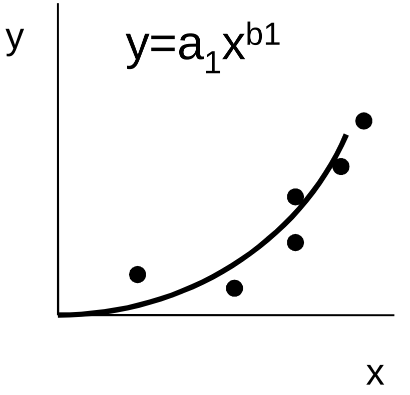

a) korelacja liniowa dodatnia ($r_{xy} > 0$)

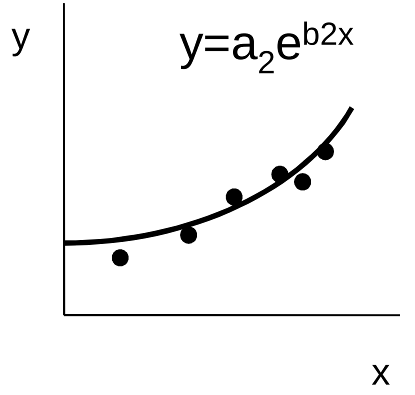
b) korelacja liniowa ujemna ($r_{xy} < 0$)

c) brak korelacji ($r_{xy} = 0$)

d) korelacja krzywoliniowa ($r_{xy} = 0$)

# Aproksymacja

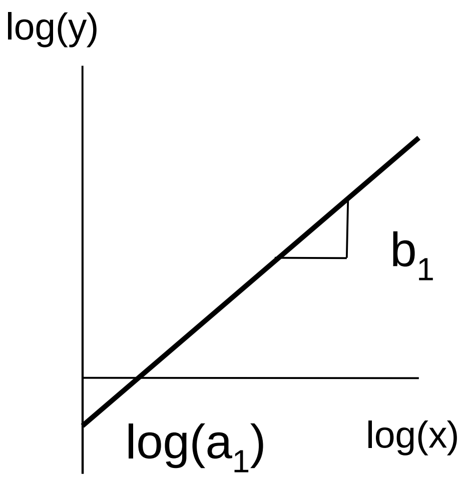


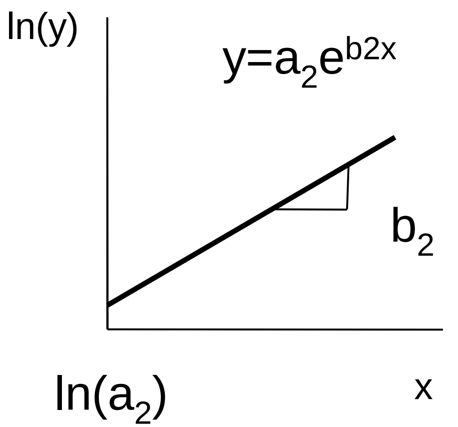




$\log(y)=b_1*\log(x) + \log(a_1)$

$\ln(y)=\ln(a_2) + b_2x*\ln(e)$

$\ln(y)= b_2x + \ln(a_2)$



# Aproksymacja

$$f(x) = c_1 x^{n-1} + c_2 x^{n-2} + \ldots + c_n x^0$$

$$f(x) = ax + b$$

$$f(x) = c_1 x^2 + c_2 x + c_3$$

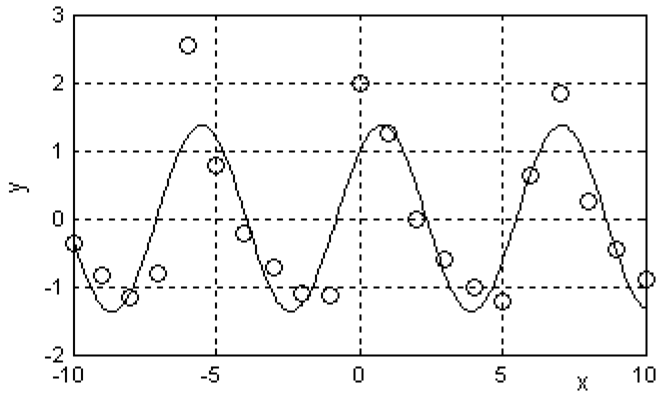
$$\boldsymbol{A} = \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \vdots \\ x_m & 1 \end{bmatrix}$$

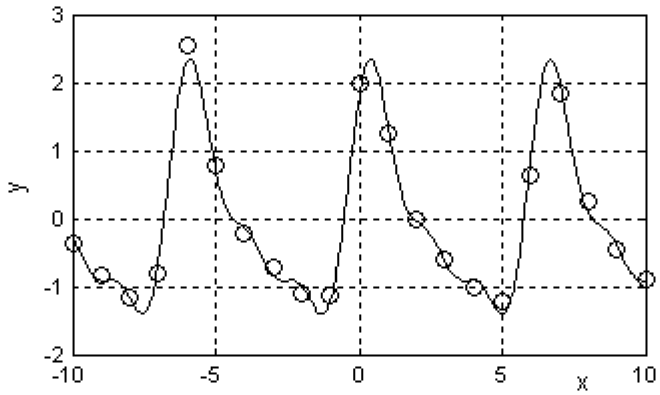
$$\boldsymbol{A} = \begin{bmatrix} x_1^{\ 2} & x_1 & 1 \\ x_2^{\ 2} & x_2 & 1 \\ \vdots & \vdots & \vdots \\ x_m^{\ 2} & x_m & 1 \end{bmatrix}$$

$$f_i(x) = x^{n-i}$$

$$f(x) = c_1 f_1(x) + c_2 f_2(x) + \ldots + c_n f_n(x)$$

# Aproksymacja




$$f(x) = c_1 f_1(x) + c_2 f_2(x) + \ldots + c_n f_n(x)$$

$f(x) = c_1\sin(x)+c_2\cos(x)$

A = [sin(x) cos(x)];

$f(x) = c_1\sin(x)+c_2\cos(x)+ c_3\sin(2x)+c_4\cos(2x)+ c_5\sin(3x)+c_6\cos(3x)$

A = [sin(x) cos(x) sin(2*x) cos(2*x)...
sin(3*x) cos(3*x)];

# Aproksymacja

$$(x_1, y_1, z_1), (x_2, y_2, z_2), \ldots, (x_n, y_n, z_n)$$

$$p_1 + p_2 x + p_3 y$$

$$S(p_1, p_2, p_3) = \sum_{i=1}^{n} \left( p_1 + p_2 x_i + p_3 y_i - z_i \right)^2 = \min$$

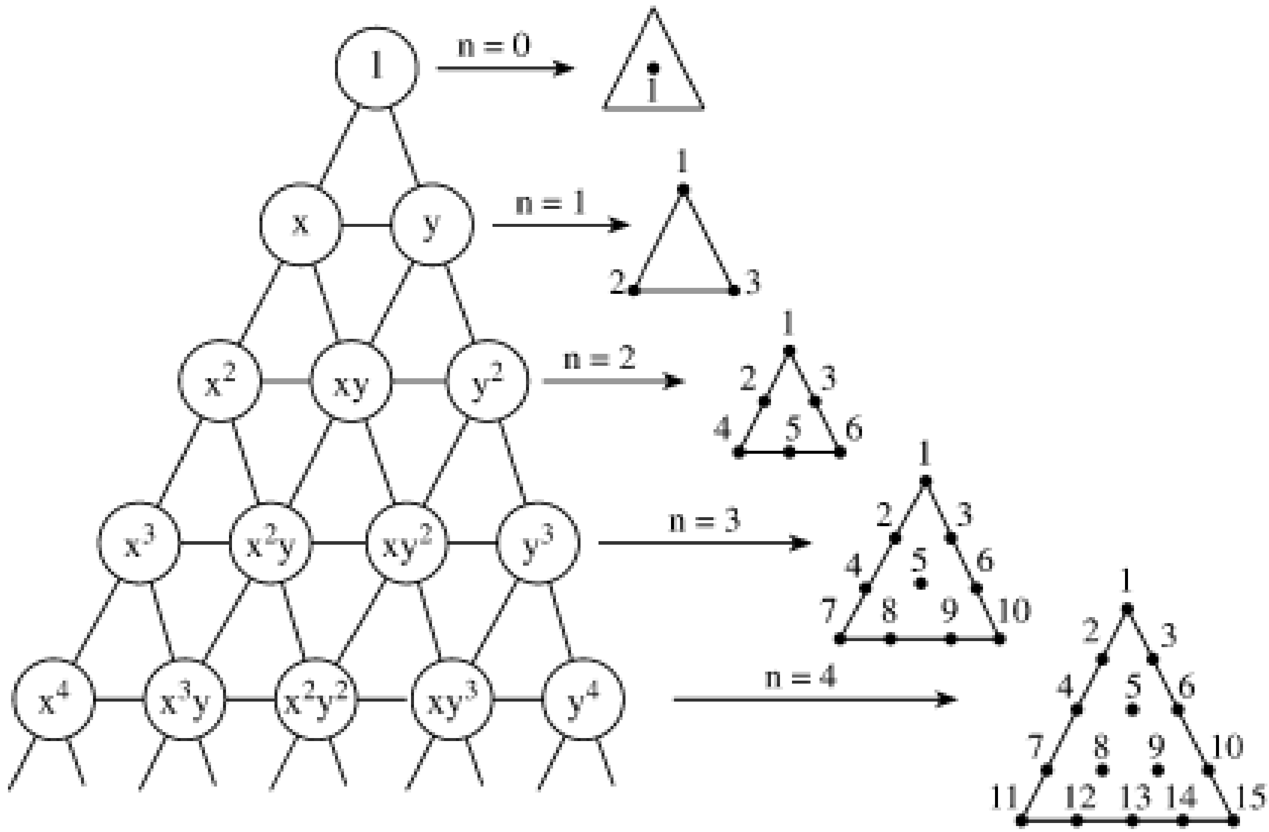
# Aproksymacja

$$\begin{bmatrix} n & \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} x_i y_i \\ \sum_{i=1}^{n} y_i & \sum_{i=1}^{n} x_i y_i & \sum_{i=1}^{n} y_i^2 \end{bmatrix} \cdot \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} z_i \\ \sum_{i=1}^{n} x_i z_i \\ \sum_{i=1}^{n} y_i z_i \end{bmatrix}$$

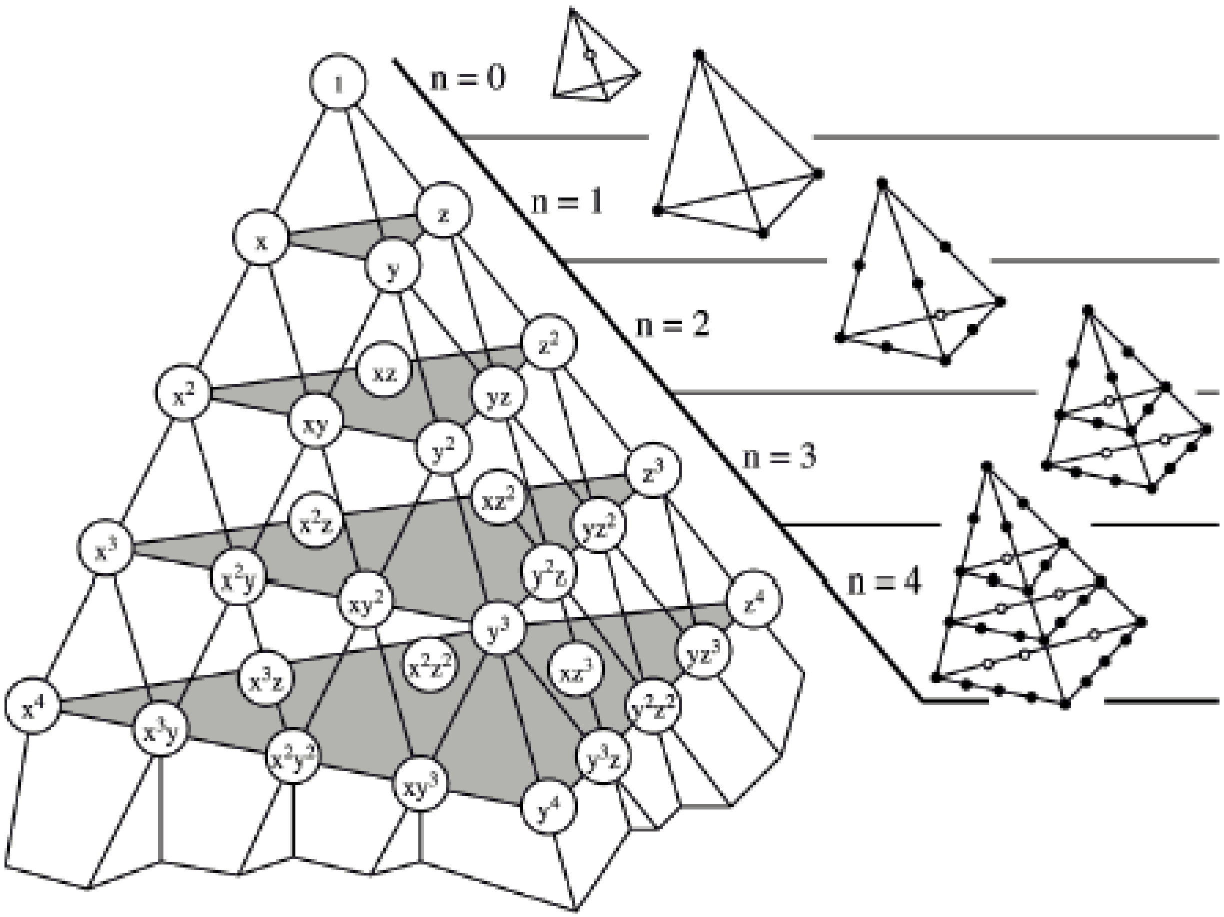
A = [1 x y];

# Aproksymacja

A = [1 x y];

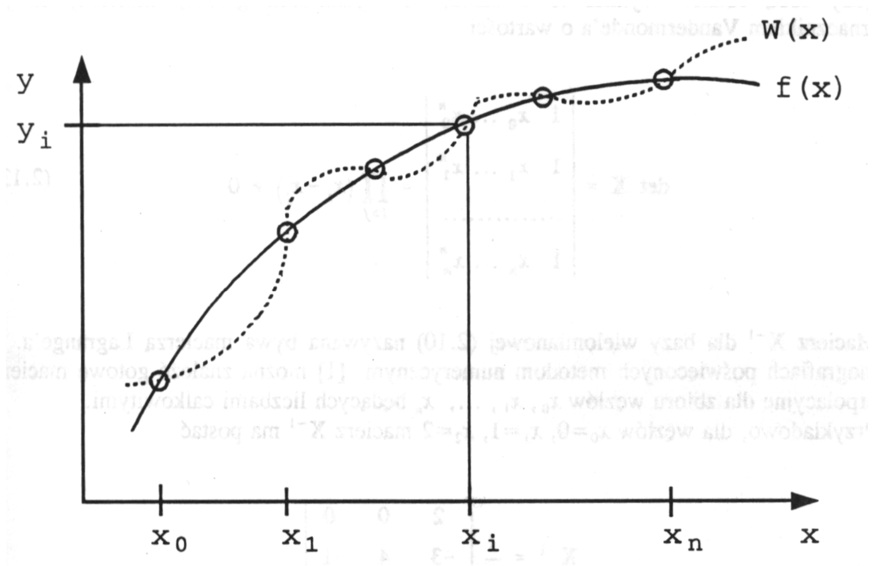


# Aproksymacja




A=[1 x y $x^2$ $y^2$ xy $x^3$ $y^3$ $x^2$y $xy^2$];

# Interpolacja




$$W(x) = c_1 x^2 + c_2 x + c_3$$

$$W(x_1) = c_1 x_1^2 + c_2 x_1 + c_3$$

$$W(x_2) = c_1 x_2^2 + c_2 x_2 + c_3$$

$$W(x_3) = c_1 x_3^2 + c_2 x_3 + c_3$$

$$\begin{bmatrix} x_1^2 & x_1 & 1 \\ x_2^2 & x_2 & 1 \\ \vdots & \vdots & \vdots \\ x_n^2 & x_n & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} W(x_1) \\ W(x_2) \\ W(x_3) \end{bmatrix}$$

# Interpolacja

$$W_2(x) = c_1 + c_2(x - x_1) + c_3(x - x_1)(x - x_2)$$

$$W_2(x_1) = c_1 = y_1$$

$$W_2(x_2) = c_1 + c_2(x_2 - x_1) = y_2$$

$$W_2(x_3) = c_1 + c_2(x_3 - x_1) + c_3(x_3 - x_1)(x_3 - x_2) = y_3$$

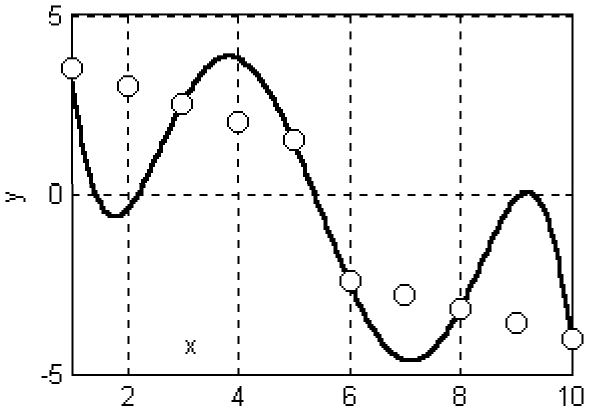
$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & (x_2 - x_1) & 0 \\ 1 & (x_3 - x_1) & (x_3 - x_1)(x_3 - x_2) \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

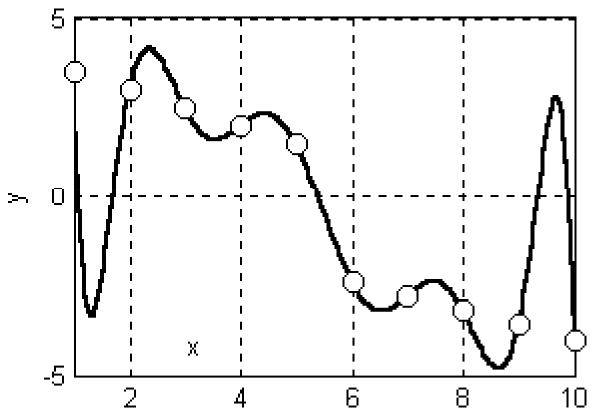
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ f[x_1, x_2] \\ f[x_1, x_2, x_3] \end{bmatrix}$$

$$f[x_1, x_2] = \frac{y_2 - y_1}{x_2 - x_1}$$

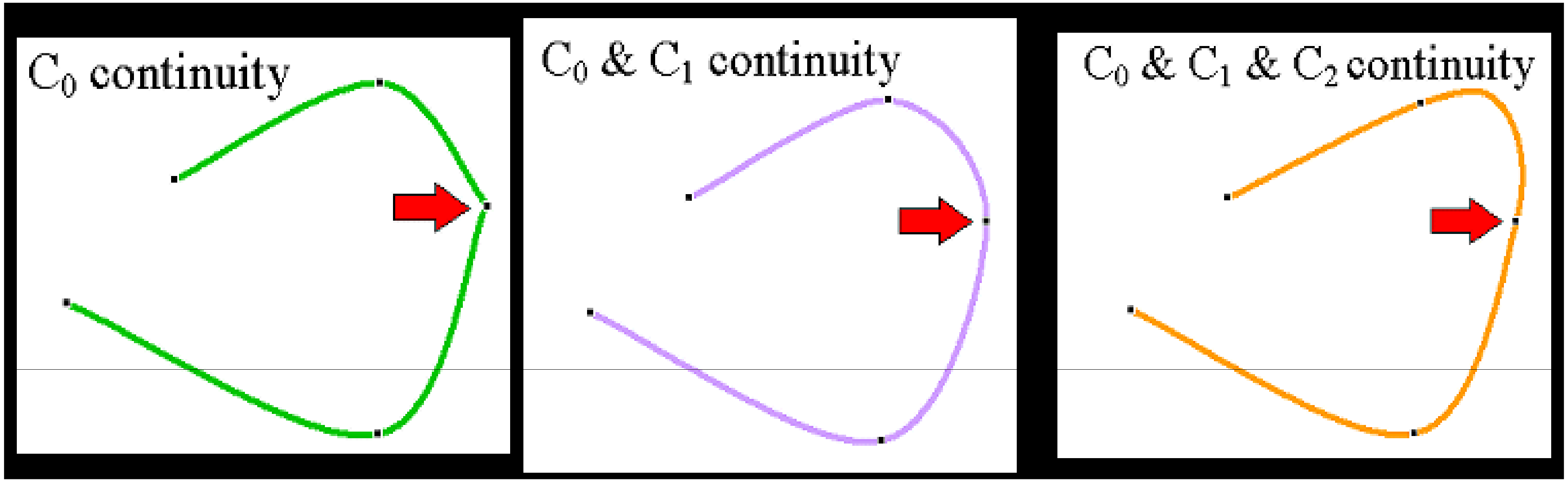
$$f[x_1, x_2, x_3] = \frac{f[x_2, x_3] - f[x_1, x_2]}{x_3 - x_2}$$

# Interpolacja



# Interpolacja




Rodzaje ciągłości:
$C_o$ - połączenie końców segmentów
$C_1$ – taka sama pochodna (styczna )
$C_2$ - ciągła 2-ga pochodna

# Interpolacja

$$x = a_x t^3 + b_x t^2 + c_x t + d_x$$

$$y = a_y t^3 + b_y t^2 + c_y t + d_y$$

$$\begin{bmatrix} x & y \end{bmatrix} = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \begin{bmatrix} a_x & a_y \\ b_x & b_y \\ c_x & c_y \\ d_x & d_y \end{bmatrix}$$

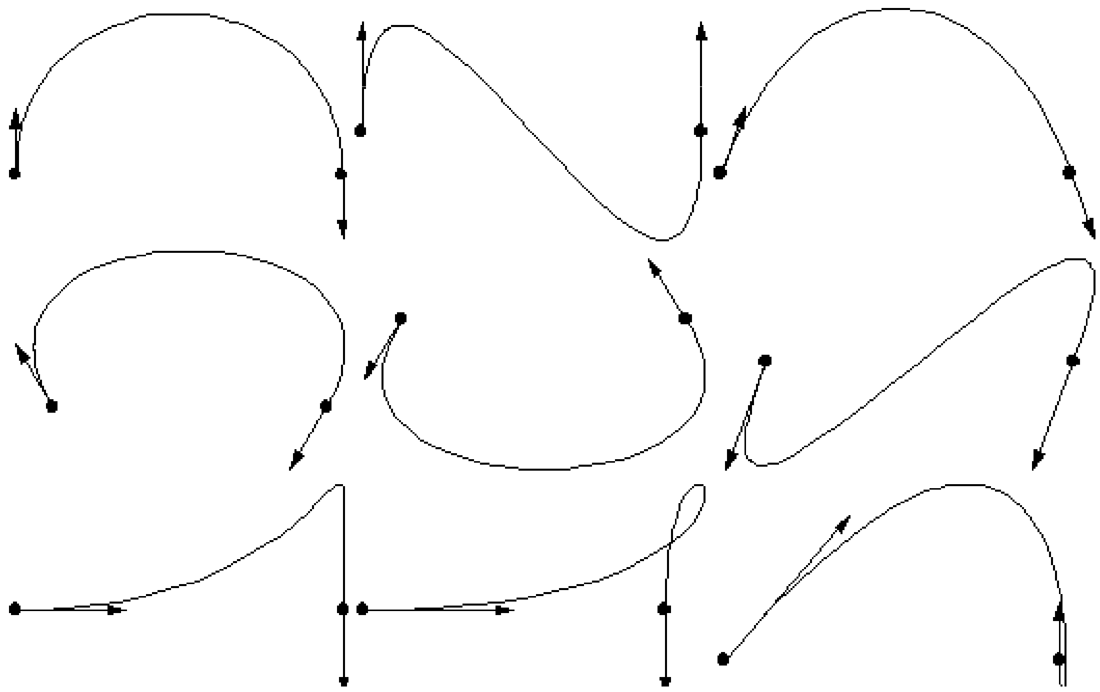
# Interpolacja

$$x = a_x t^3 + b_x t^2 + c_x t + d_x$$

$$y = a_y t^3 + b_y t^2 + c_y t + d_y$$

$$[x \quad y] = [t^3 \quad t^2 \quad t \quad 1]\begin{bmatrix} a_x & a_y \\ b_x & b_y \\ c_x & c_y \\ d_x & d_y \end{bmatrix}$$

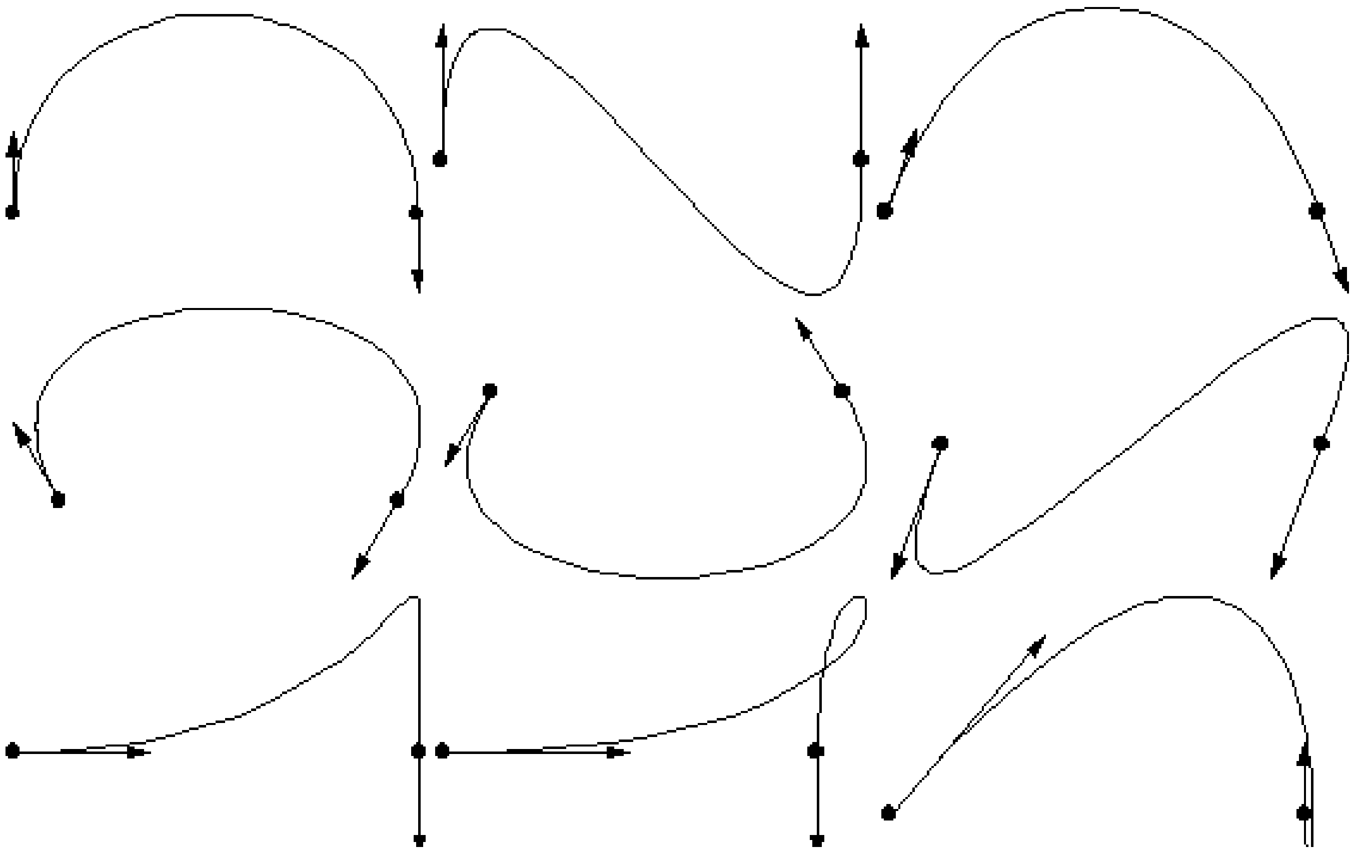
# Interpolacja

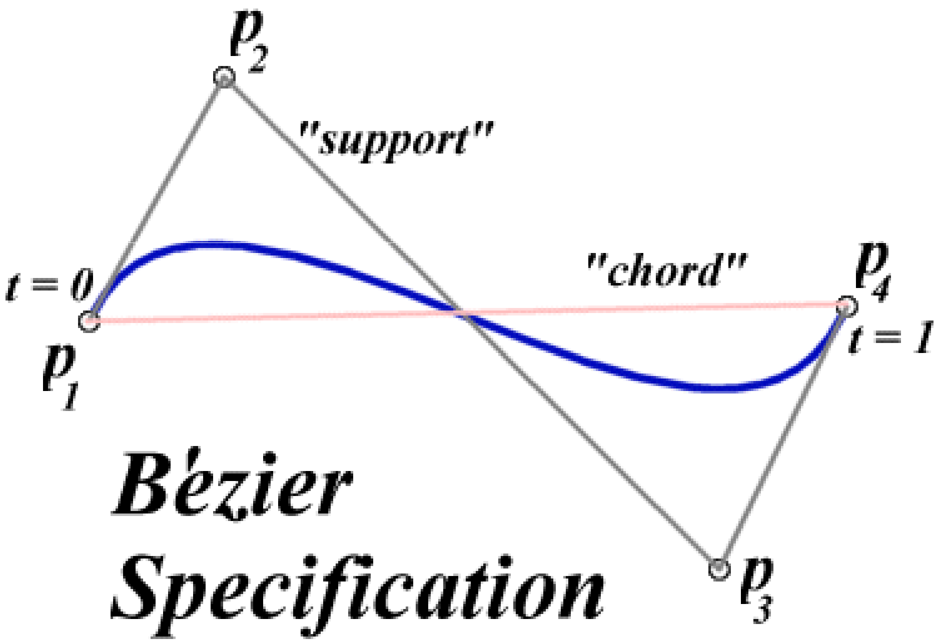
# Interpolacja


$$[x \quad y] = [t^3 \quad t^2 \quad t \quad 1] \underbrace{\begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}}_{\mathbf{M}_{Hermite}} \underbrace{\begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ \frac{dx_1}{dt} & \frac{dy_1}{dt} \\ \frac{dx_2}{dt} & \frac{dy_2}{dt} \end{bmatrix}}_{\mathbf{G}_{Hermite}}$$

# Interpolacja



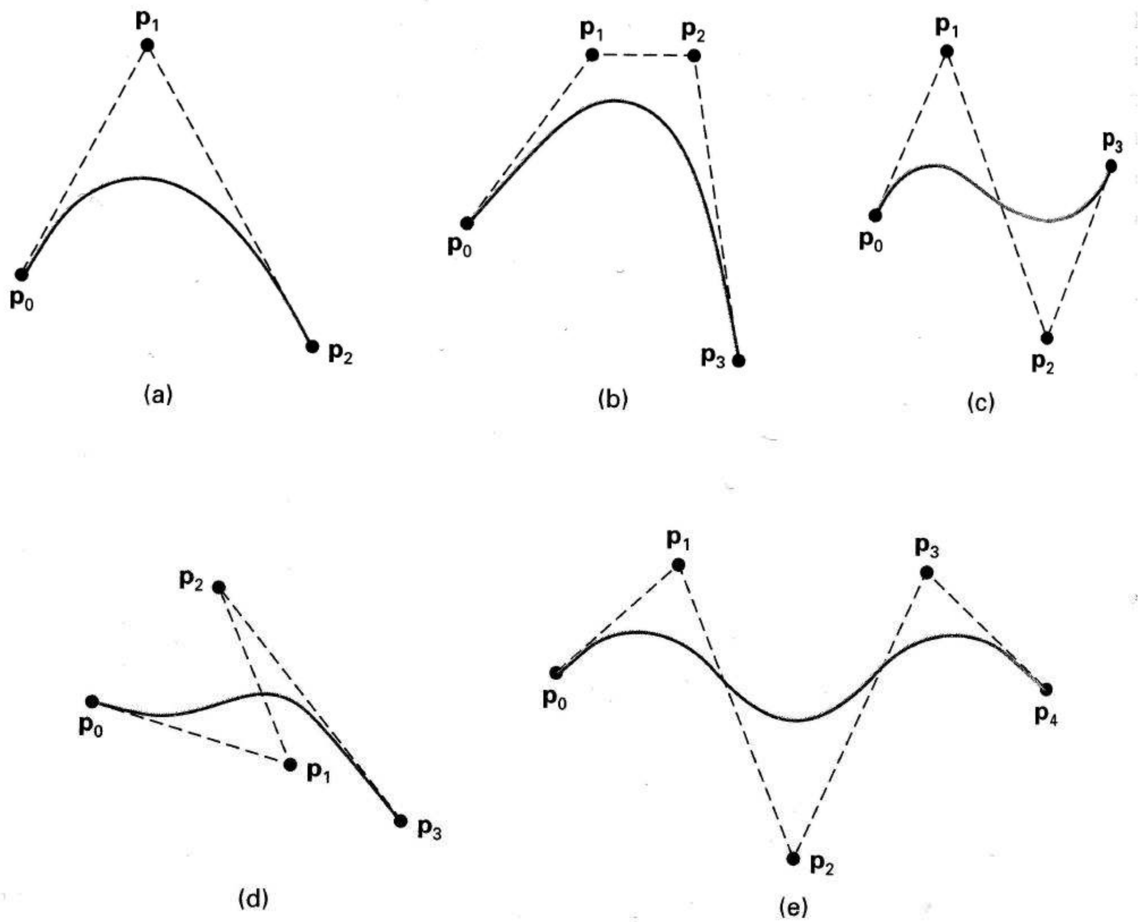
# Interpolacja




$$\nabla p_1 = 3(p_2 - p_1)$$

$$\nabla p_4 = 3(p_4 - p_3)$$

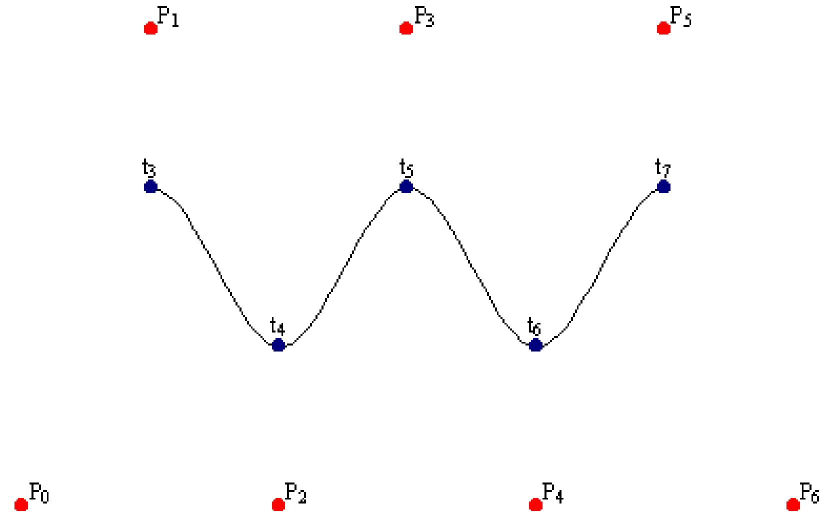
# Interpolacja

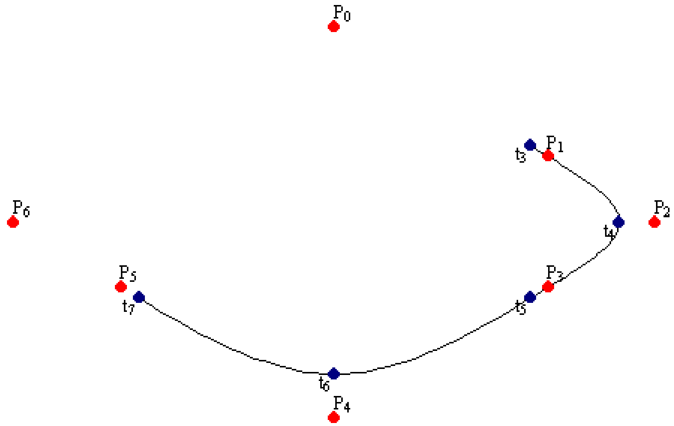


# Interpolacja

- Krzywe Beziera i Hermite’a są definiowane globalnie
  - Kawałkami sklejane (piecewise) krzywe Beziera i Hermite’a nie gwarantują ciągłości pochodnych na łączeniach
  - Przesunięcie jednego punktu kontrolnego zmienia cała krzywą

- B-splajny składają się z fragmentów krzywych, których współczynniki zależą tylko od kilku punktów kontrolnych – są definiowane lokalnie



# Interpolacja